

WFIRST Planet Masses from Microlens Parallax

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ABSTRACT

I present a method using only a few ground-based observations of a magnified microlensing event to routinely measure the parallaxes of WFIRST events if WFIRST is in an L2 orbit. This could be achieved with target-of-opportunity observations of select WFIRST events, but is better done with a complementary, ground-based survey of the WFIRST fields. Such a survey would measure the parallax for essentially all events with $A_{\max} \gtrsim 30$. When combined with a measurement of the angular size of the Einstein ring, which is almost always measured in events with planets, these parallax measurements will routinely give measurements of the lens masses and hence, the absolute masses of the planets. It can also lead to mass measurements for dark, isolated objects such as brown dwarfs, free-floating planets, and stellar remnants if the size of the Einstein ring can be determined.

Subject headings: gravitational lensing:micro, planets and satellites: general

1. Introduction

The microlensing portion of the WFIRST mission will complete the census of planets by finding large populations of planets beyond the snow line with masses as small as that of Mars (Green et al. 2012). If the masses of the planets and their hosts are measured, this will permit a direct comparison to planet formation theories. However, the primary observables in a microlensing event are the mass ratio and projected separation (scaled to the Einstein ring) between the planet and its host star. A measurement of the lens mass is necessary to transform these to physical quantities.

If the lens is bright enough, WFIRST will be able to estimate its mass based on a measurement of the lens flux. However, there will be many cases for which the lens light will be too faint to be measured. Such cases will most likely be lenses at the bottom of the stellar mass function, but could also include lenses that are brown dwarfs, free-floating planets, or stellar remnants. For these events, with only an upper limit on the lens flux, the conclusions that can be drawn about the nature of the planet are limited. In addition, the WFIRST measurement of the lens flux will be a measurement of the total flux of the lens system, including any companions to the lens, which may or may not

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participate in the lensing event. Typically, companions within 10 AU will produce a measurable microlens perturbation and companions at more than 5 mas (~ 40 AU) will be identifiable from a shift in the centroid relative to the lensing event. However, companions at intermediate separations are not easily identified. Hence, the WFIRST mass estimates will necessarily be upper limits for any given system.

Fortunately, the masses of the lenses can be measured if microlens parallax and finite source effects are observed. Microlens parallax is a vector quantity whose magnitude is the size of the Earth’s orbit (AU) compared to the size of the Einstein ring projected onto the observer plane, \tilde{r}_E :

$$\pi_E \equiv \frac{\text{AU}}{\tilde{r}_E}. \quad (1)$$

If π_E is measured, the mass of the lens can be obtained with a measurement of the angular size of the Einstein ring. This is possible for any event in which the size of the source is resolved in time, i.e., it passes over a caustic or near a cusp. Finite source effects are almost always measured in events with planets because detection of a planetary companion to the lens almost always requires a caustic interaction. Hence, if the microlens parallax can be measured, the masses of the planets are known. Finite source effects can also be measured in any event for which the source crosses the position of the lens.

In this paper, I discuss a means to routinely measure the lens masses using microlens parallax if WFIRST is in an L2 orbit¹. Because the WFIRST light curve will be measured so precisely, the orbital parallax effect will be routinely detected at high significance, effectively giving one extremely well-measured component of the parallax (Gould 2013). I show that only a few ground-based observations of each event are needed to complement the WFIRST observations and yield a complete parallax measurement for a large fraction of events.

2. Measuring $\pi_{E,\perp}$

2.1. A Simplified Case

The microlens parallax vector can be written

$$\boldsymbol{\pi}_E = (\pi_{E,\parallel}, \pi_{E,\perp}) = (\pi_E \cos \theta, \pi_E \sin \theta), \quad (2)$$

where θ is the angle between the lens trajectory and the projection of the Sun-Earth line on the plane of the sky, measured counter-clockwise. Because WFIRST will be in orbit about the Sun, there will be a measurable asymmetry in the light curve due to the orbital parallax effect (Gould 1992; Gould et al. 1994). This gives strong constraints on the component of the parallax parallel to

¹See Gould (2013) for a discussion of WFIRST parallax measurements for a geocentric orbit.

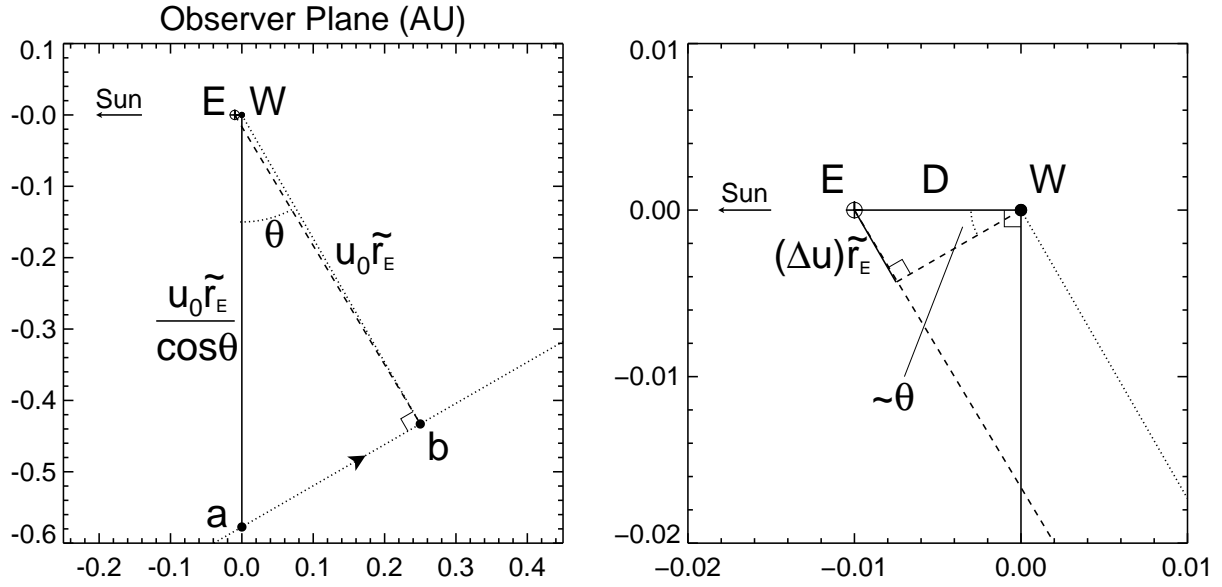


Fig. 1.— Left panel: the basic geometry of a microlensing event projected onto the observer plane. The right-hand panel expands the view around the projected positions of the Earth and WFIRST on this plane (‘E’ and ‘W’, respectively). The x-axis is East-West and the y-axis is North-South. The dotted line ab with the arrow shows the lens trajectory. The value of $\pi_{E,\perp}(\text{AU})^{-1} = \sin \theta / \tilde{r}_E = \Delta u / D$ can be derived from the observables Δu and D , while $\pi_{E,\parallel} = \cos \theta / \tilde{r}_E$ can be measured by WFIRST alone.

the projected position of the Sun relative to the event, $\pi_{E,\parallel}$, but usually only very weak constraints on the other component, $\pi_{E,\perp}$ (e.g., Gould 2013).

I will show that if WFIRST is at L2, as few as two observations of a microlensing event from the Earth can be used to measure $\pi_{E,\perp}$, leading to a measurement of π_E . To begin with, I will start with a simplified case to illustrate the problem. In the following section, I will present the full derivation and expression for $\pi_{E,\perp}$ and show that it reduces to what is derived here.

Consider the projection of the microlensing event onto the observer plane (Fig. 1). For the purposes of illustration, I have assumed that the size of Einstein ring projected onto this plane is $\tilde{r}_E = 10$ AU, $u_0 = 0.05$, and that the observations are taken close to the equinox (the anticipated midpoint of WFIRST observations) so that the projection of the Earth-WFIRST line onto the sky, D , is equal to the true separation, i.e., $D = 0.01$ AU. The exact values for these quantities are irrelevant to the derivation; I will discuss their practical implications below. Finally, note that WFIRST at L2 puts it in line with the Earth and Sun, so the projection of the Earth-WFIRST line on the sky is parallel to the projection of the Sun’s position.

The WFIRST light curve will be extremely well measured, giving $\pi_{E,\parallel}$ and the basic microlens parameters: the time of the peak, the source-lens impact parameter scaled to the Einstein ring, and the Einstein crossing time (t_0 , u_0 , and t_E , respectively). Hence, the value of $u_0 \tilde{r}_E (\cos \theta)^{-1}$ is also known. This fixes point ‘a’ on the lens trajectory projected onto the observer plane. Assume the event is observed from the Earth when it is at the peak as seen from WFIRST (i.e., when the lens is at point ‘b’). Then, the fractional difference in the magnification is given by

$$\frac{\Delta A}{A} = \frac{A - A_{\oplus}}{A} \simeq \frac{\Delta u}{u_0} \quad (3)$$

where A is the magnification as seen from WFIRST, A_{\oplus} is the magnification as seen from the Earth, and I have assumed that the magnification is given by $A \simeq u_0^{-1}$ (which applies in the limit $u_0 \ll 1$) and that the difference between the impact parameter as seen from Earth and from WFIRST is $\Delta u \ll u_0$. As illustrated in Figure 1, in the regime where $u_0 \tilde{r}_E \gg D$, $(\Delta u) \tilde{r}_E \simeq D \sin \theta$ meaning that with some manipulation $\pi_{E,\perp}$ can be written:

$$\pi_{E,\perp} = u_0 \left(\frac{\Delta A}{A} \right) \left(\frac{\text{AU}}{D} \right). \quad (4)$$

Note that all of the variables in the right-hand side of the equation are known or measurable.

The uncertainties from the WFIRST light curve are negligible compared to the uncertainties from the ground-based photometry, so u_0 and A are measured essentially perfectly. The largest uncertainty in $\pi_{E,\perp}$ comes from the measurement of $(\Delta A)A^{-1}$. The actual observables from the Earth are the magnified flux, $f_{\text{mag},\oplus}$, and flux of the event at the baseline, $f_{\text{base},\oplus}$, such that

$$A_{\oplus} = \frac{f_{\text{mag},\oplus} - f_{\text{base},\oplus}}{f_{\text{S},\oplus}} + 1, \quad (5)$$

where $f_{\text{S},\oplus}$ is the flux of source as seen from the Earth and can be estimated by calibrating the ground-based photometry to the WFIRST photometry. Based on previous experience (e.g., Yee et al. 2012), the uncertainty in this calibration is limited by systematics to a precision of about 1%. This sets the fundamental noise floor on the measurement of $(\Delta A)A^{-1}$. Given that by definition, $|(\Delta u)\tilde{r}_{\text{E}}| \leq D$, the limit in the flux precision means that a 3σ measurement of $\pi_{\text{E},\perp}$ is possible for

$$u_0 \leq 0.03 \left(\frac{D}{0.01 \text{AU}} \right) \left(\frac{\tilde{r}_{\text{E}}}{10 \text{AU}} \right)^{-1} \left(\frac{\sigma_{\Delta A/A}}{0.01} \right)^{-1} \quad (6)$$

where I have again made the assumption that $u_0 \ll 1$, i.e., $A \simeq u_0^{-1}$.

In addition to the measurement uncertainty in the magnitude of $\pi_{\text{E},\perp}$, there is also a degeneracy in its sign. Based the geometry given in Figure 1, there is one degeneracy in Equation (4). It is possible to change the sign of u_0 , i.e., reflect the figure over the x-axis, which changes the sign of $\pi_{\text{E},\perp}$. This leads to a degeneracy in the direction of $\boldsymbol{\pi}_{\text{E}}$, but not in its magnitude, which is the relevant quantity for calculating masses. Out of the eight possible configurations one might consider as potentially degenerate with the geometry shown, only the $u_0 \rightarrow -u_0$ degeneracy described here is permitted by the observables².

Finally, I note that although the measurement of $\pi_{\text{E},\perp}$ using this method will not be as good as the measurement of $\pi_{\text{E},\parallel}$ from the orbital parallax, this does not mean that the value of π_{E} is not well measured. So long as $\pi_{\text{E},\parallel} \gtrsim 3\sigma_{\pi_{\text{E},\perp}}$, the constraints on π_{E} will be useful. This will be true for a large fraction of cases, depending on how well the lens trajectory aligns with the projection of the Sun-Earth-WFIRST line, which is primarily a random effect.

2.2. The Exact Expression for $\pi_{\text{E},\perp}$

I now derive a general expression for $\pi_{\text{E},\perp}$ that applies for an Earth-based measurement of the magnification at any time (rather than one taken precisely at the peak of the event). In practice, this is the expression that will be used to calculate $\pi_{\text{E},\perp}$ from the observables.

Figure 2 shows the geometry for this general case. To simplify the figure labels, I have scaled all quantities to \tilde{r}_{E} . Consider that an observation from the Earth is taken at time t , i.e., when the lens is at point ‘C.’ Its position is given by u_0 and $\tau = (t - t_0)t_{\text{E}}^{-1}$. The measured magnification is related to the separation of the lens, u , by

$$A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}, \quad (7)$$

where u is measured as a fraction of the Einstein ring. Thus, from the measured magnifications as seen from the Earth and WFIRST, the lens separation from their projected positions is known, u_{E}

²Note that because of the special geometry i.e., WFIRST is in line with the Earth-Sun line, the u_0 degeneracy in $\boldsymbol{\pi}_{\text{E}}$ from Dong et al. (2007) does not apply.

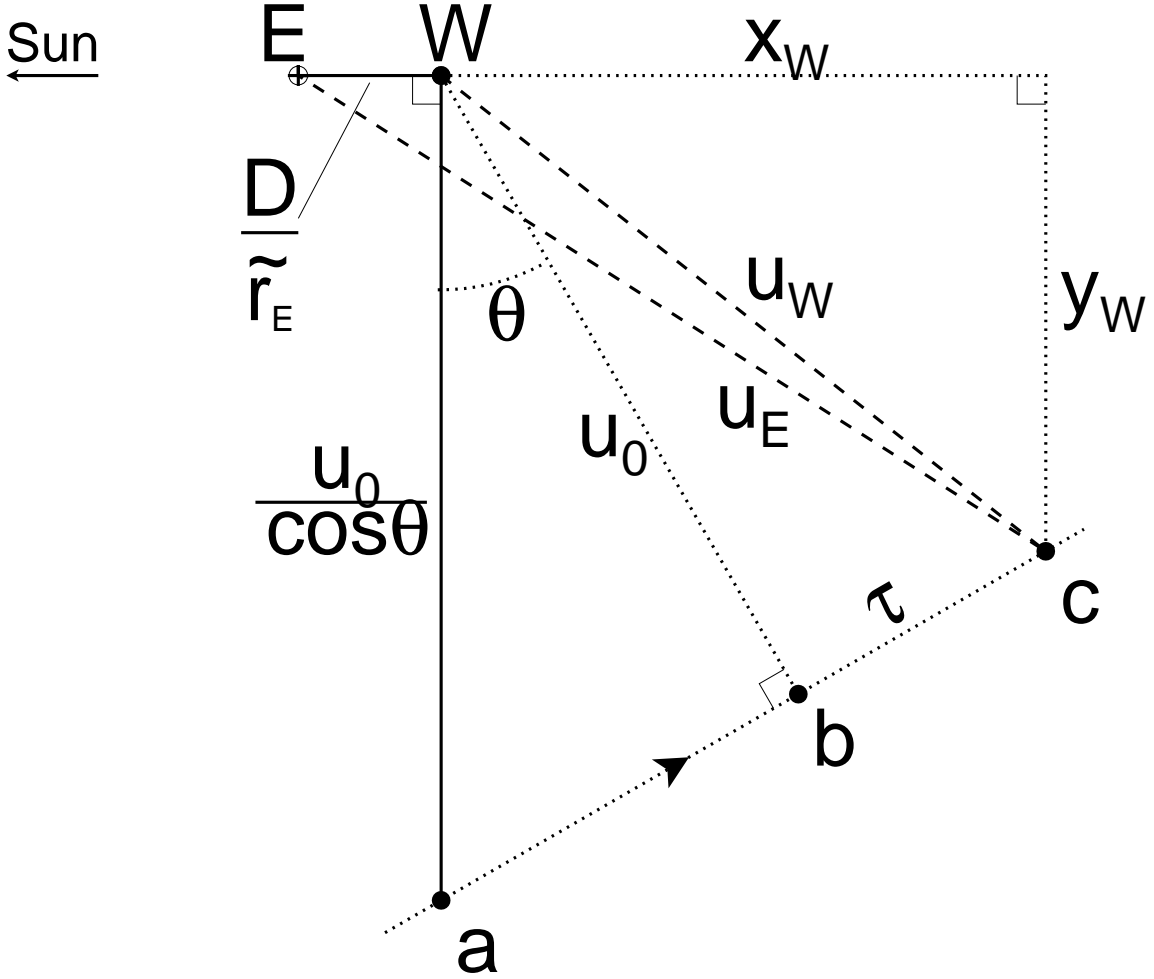


Fig. 2.— Generalized geometry of a microlensing event projected onto the Observer planet. This figure is analogous to Fig. 1 except that for simplicity, all quantities are scaled to the size of the Einstein ring, \tilde{r}_E . The line abc indicates the trajectory of the lens. The projected positions of the Earth and WFIRST are labeled ‘E’ and ‘W’, respectively.

and u_W , respectively. I can then write

$$u_E^2 = (x_W + D/\tilde{r}_E)^2 + y_W^2, \quad (8)$$

where x_W is the projection of u_W onto the x-axis (the Earth-WFIRST line) and y_W is the projection of u_W onto the y-axis. Equation (8) can be rewritten as

$$u_E^2 - u_W^2 = \frac{2D}{\tilde{r}_E}(u_o \sin \theta + \tau \cos \theta) + \frac{D^2}{\tilde{r}_E^2}. \quad (9)$$

Recognizing that $(\pi_{E,\parallel}, \pi_{E,\perp}) = (\cos \theta, \sin \theta)(\text{AU})\tilde{r}_E^{-1}$ (Eqn. 1 and 2), I can then write

$$\pi_{E,\perp} = \left[\Delta u(2u_W + \Delta u) - \frac{D^2}{\tilde{r}_E^2} \right] \left(\frac{\text{AU}}{2Du_0} \right) - \frac{\tau \pi_{E,\parallel}}{u_0}, \quad (10)$$

where $\Delta u \equiv u_E - u_W$. Given that $\tilde{r}_E^{-2} = (\pi_{E,\perp}^2 + \pi_{E,\parallel}^2)(\text{AU})^{-2}$, this can be rewritten as a quadratic equation for $\pi_{E,\perp}$ with the solutions:

$$\pi_{E,\perp,\pm} = u_0 \left(\frac{\text{AU}}{D} \right) \left(-1 \pm \sqrt{1 - \frac{1}{u_0^2} \left[\left(\frac{D}{\text{AU}} \right)^2 \pi_{E,\parallel}^2 + 2 \left(\frac{D}{\text{AU}} \right) \tau \pi_{E,\parallel} - \Delta u(2u_W + \Delta u) \right]} \right). \quad (11)$$

Although there are formally two solutions for $\pi_{E,\perp}$, these can readily be distinguished. The solution $\pi_{E,\perp,-}$ corresponds to the case in which the lens passes between the projected positions of the Earth and WFIRST. This solution is pathological and is expected to be very rare, but it can be definitively excluded with additional observations of the event from the Earth.

If π_E is large, then the full expression must be evaluated. However, the present paper is primarily interested in cases for which π_E is small because those are the cases in which the WFIRST light curve will constrain only one component of the parallax well. In that case, Equation (11) is well represented by the first term in the Taylor expansion:

$$\pi_{E,\perp,+} = \frac{1}{2u_0^2} \left[\Delta u(2u_W + \Delta u) - 2 \left(\frac{D}{\text{AU}} \right) \tau \pi_{E,\parallel} - \left(\frac{D}{\text{AU}} \right)^2 \pi_{E,\parallel}^2 \right]. \quad (12)$$

Since the case under consideration is one for which π_E is small, the last term can generally be ignored because it is second order. Finally, if we consider the case that the event is observed at peak ($\tau \rightarrow 0$ and $u_W \rightarrow u_0$) and assume that $|\Delta u| \ll |u_0|$, then

$$\pi_{E,\perp} \rightarrow \Delta u \left(\frac{\text{AU}}{D} \right), \quad (13)$$

which is equivalent to Equation (4).

3. Discussion

I have shown that for an event discovered by WFIRST at L2, a measurement of the event magnification as seen from Earth can give a measurement of the component of the parallax perpendicular to the projection of the Earth’s orbit, $\pi_{E,\perp}$. Although the basic calculation was done assuming the event was observed from the ground at the peak as seen from WFIRST, I showed that in principle the Earth measurement can be made at any time. However, it is best to make the measurement as close to the peak of the event as possible, since the fractional difference in magnification will be largest at the peak, allowing for the best measurement of the parallax. Measuring the magnification of the event as seen from Earth requires at least two observations: one when the event is magnified and one when the event is at baseline. A third, magnified, observation would be beneficial in case the source is passing over a caustic at the time of the observations and for distinguishing between the two possible solutions for $\pi_{E,\perp}$.

The method presented here is qualitatively similar to what was presented in Gould & Yee (2013) for measuring parallaxes for high-magnification events (peak magnification $A_{\max} \gtrsim 100$) seen from the ground using three observations from a satellite. However, there are several notable differences. Most prominent is the fact that one component of the parallax will be essentially perfectly measured from the WFIRST light curve. Because of this, all of the degeneracies affecting the magnitude of the parallax (Gould 1994) can be resolved without requiring that the event be high magnification as in Gould & Yee (2012). In addition, the parallax measurement is possible despite a much shorter baseline (0.01 AU vs. 1 AU) than considered in Gould & Yee (2012).

The ground-based magnification measurement could be done as a target-of-opportunity observation from the Earth if WFIRST issued real-time information on ongoing microlensing events. If only a few events can be observed from the ground, it is best to focus on the highest magnification events. I have shown that if the flux can be measured to 1% precision from the ground, $\pi_{E,\perp}$ is measurable in all events with $u_0 \tilde{r}_E < 0.3\text{AU}$, i.e., $u_0 < 0.03$. In practice many of the WFIRST sources will be quite faint, so while 1% precision will be possible for the brighter sources, the limits on u_0 are probably more stringent for the majority of events. This leads to a preference for higher magnification events. However, it is precisely these events for which parallax measurements are most desirable because these events are the most likely to have planetary signals (e.g., Yee et al. 2009; Abe et al. 2013).

Furthermore, for point lens events, the higher the magnification, the more likely it is that finite source effects will be observed, allowing mass measurements for these lenses. These isolated objects could include stellar remnants, isolated stellar mass black holes, brown dwarfs, or the population of free-floating planets found by Sumi et al. (2011). Gould & Yee (2013) also proposed a means to measure the mass of free-floating planets using terrestrial parallax. However, because the baseline for terrestrial parallax measurements is $\sim R_\oplus$, parallaxes are only measurable for the closest objects. Here, parallax measurements are possible for a much more distant lenses (and hence, a larger volume and larger number of events) because of the larger Earth-WFIRST baseline.

Hence, targeted observations for measuring parallaxes should focus on the higher magnification events.

However, a NIR, ground-based survey simultaneous with WFIRST is a superior approach relative to target-of-opportunity observations. In the case of events with planets, 50% of the time there will not be advanced warning that a parallax observation is needed because the planetary perturbation will occur after the peak of the event. A survey has the advantages that it would not require real-time information from WFIRST, the parallax would be measured for *all* events for which it was possible, and multiple points would be taken throughout the light curve, improving the precision of the measurement.

Although the method for obtaining parallaxes described here is observationally intensive, the potential scientific impact makes such observations invaluable. If masses are measured for the lens stars, this provides a direct test of the core accretion theory of planet formation, which predicts that giant planets should be rare around typical microlensing hosts, which are M dwarfs (Laughlin et al. 2004; Ida & Lin 2005). Furthermore, when the lens masses are measured, so are their distances. Such measurements would allow a comparison of the planet populations in the bulge and the disk. Given that the stars (and planets) in the bulge formed in a dense region of rapid star formation, one might expect a dearth of giant planets there (Thompson 2013). Finally, if microlens parallax is measured for many events, including ones with lens mass estimates from WFIRST, this will allow the first systematic test of microlens parallax.

In addition, although WFIRST will measure the lens system fluxes for many events, whether the light comes from the lens itself or a stellar companion will be unknown. Systematic measurements of the microlens parallax can be used to measure the fraction of events for which the lens light is contaminated by the presence of a companion not involved in the microlensing event.

Finally, although this paper has been written from the perspective of the WFIRST mission, it is broadly applicable to any microlensing satellite at L2. Thus, if a microlensing survey is included in the Euclid mission, it would also benefit from complementary ground-based observations. In fact, for Euclid a ground-based parallax campaign is even more important for measuring the lens masses because its NIR resolution will make it more difficult to accurately measure the lens system fluxes.

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